

LICHHAVI NUMERAL SYSTEMS IN NEPAL

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Abstract: Numerals and numeral systems, a collection of symbols used to represent small numbers, together with a system of rules for representing larger numbers. These systems plays a vital role in the development of mathematical science. Ancient numerals used in Nepal supports to mathematics in Nepal and hence in South Asian Sub Continent. The existence of Lichhavi numeral system is quite hidden. So for the exploration of supportive facts of declaration of Lichhavi Era, Lichhavi Dynasty and focus of the development of Lichhavi numeral system is the task of this paper in document analysis and historical base.

Keywords and Phrases: Lichhavi, Numerals, Nepal, Symbols, Maligaun.

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1. Introduction

Numbers and counting are a part of everyone's life, and understanding to numbers and their structures are the fundamental to progress in mathematics [10]. Now, due to development of fundamental elements 'numbers' and 'numerals' occurred connection among different civilisations and communities. They need exchange and made a common systems to each other. For this task, numbers and numeral systems support to society.

The history of numerical thought seems to be the way, first discover numbers, which are discrete quantities. Second, to invent physical tokens like strings, stones, bones, notches and pebbles, etc., to represent numbers. Third, people invent words and symbols to represent numbers. This last step presents the problem of numeration, how to represent numbers by words and symbols and a system of numeration

represents an attempt to solve this problem [5].

It is found that number system and counting system is one of the most important concept in mathematics. One-to-one-correspondence is a major theory in different fields of mathematics. But it was very popularly used by early, ancient and modern people to record their cattle, family members, and crops or any other data. It is also justified by the statement of Pythagoras of Samos that, “Number is within of all the things” [3]. Exploration of Lichhavi numeral system and its practices as an important event in research. The numerals, numbers and symbols are the most important in the development of mathematics in every civilization [14].

Nepal’s recorded history was began with the Kirats, who arrived in the 7th century BC to 8th century BC from the East. They were come here as sheep farmers and liking for carrying long knives. It was during this period that Buddhism first came to the country; indeed it is claimed that Buddha and his disciple Ananda visited the Kathmandu Valley and stayed for a time in Patan. Lichhavi period was known as the “Golden Period” in the history of Nepal [7]. Certain Gupta characters were used in the Nepali inscriptions [19].

Subedi (2017) highlighted the development of numerals system in ancient Nepal and its importance. The development of numeral system is the study of origin of numeral system and evolution of numeral system. Her research was attempted at the study of the process of improvement of numeral system in general situation to the advanced establishment in Nepal.

Jadranka highlighted the importance and powers of the numerals systems in linguist and he said that numbers will bear the changes of the world [8]. So numbers influenced the development for the world. Thus in this world either civilizations’ numeral system would impact, like Lichhavi numerals used for records in monuments. Here the main concern is to explore the development of numeral systems in Nepal that is the development of Lichhavi numeral system. Few evidences of numeral systems are illustrated below.

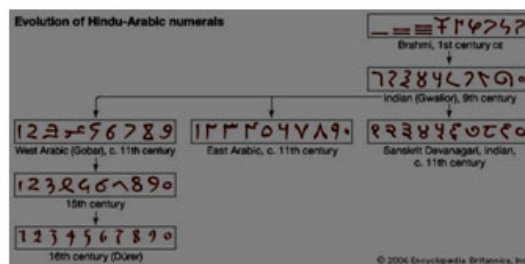


Fig. 1: Development of Different Numerals from Brahmi Numeral [22]

Tab. no.1: Lichhavi Numerals [15]

One	Two	Three	Four	Five	Six	Seven	Eight	Nine
𑒀	𑒁	𑒂	𑒃	𑒄	𑒅	𑒆	𑒇	𑒈
Ten	Twenty	Thirty	Forty	Fifty	Sixty	Seventy	Eighty	Ninety
𑒉	𑒊	𑒋	𑒌	𑒍	𑒎	𑒏	𑒐	𑒑
Hundred	Two Hundred	One Thousand						
𑒒	𑒓	𑒔						

Hui-Chih Yu explained the importance of numbers in his paper 'A Comparative Study of the Meanings of Numbers in English and Chinese Cultures' as: *the use of numbers is more than a convenient measure of the physical world. In many traditions they are linked with cosmic principles that give order and structure to the universe* [6]. On the basis of all these views discussed above that culturally and practically numbers and numerals are more influential elements in developments, so Lichhavi numeral system also impact to the development of mathematics to related communities and civilizations.

2. Objectives

This study would be explored the developments of Lichhavi numerals system, widely used to records the ancient works in different constructions, monuments like Changu Narayan Temple at Bhaktapur etc. This study would be find the number of symbols used in Lichhavi numeration system and its base system, explore the time period of the development of Lichhavi numeral system and investigate the pattern of Lichhavi numeral system.

3. Methodology

This research paper would comprised a historical document analysis of the different stages in the use of some numbers and the development of the number system in Nepal in Lichhavi period followed by an application and principles of development of numeration system.

3.1 Reviews and Analysis of Existence of Lichhavi Numerals

A few evidences of Lichhavi numerals are illustrated here 𑒀𑒉𑒅𑒆. This indicates 386. Ligatures are used to form the large numbers in Lichhavi numerals system

[12]. In a research the development of Lichhavi and Brahmi digits' symbols were illustrated as interrelated development of numeral systems [1]. Similarly the development of numerals systems in Nepal was exposed in a report "*Mathematics Education for the Twenty First Century New Nepal*" with mentioning the number 107 used by Jaya Varma which are showing as photographic form below [14].

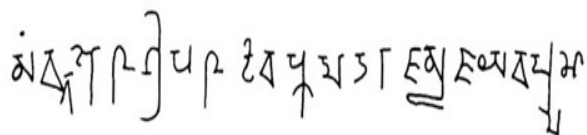


Fig. 2: Examples of Lichhavi Numerals found at Maligaun, Kathmandu

A paper "Mathematics in Nepal: A Historical Analysis" published in a journal *Scientific World* is considered as a millstone in the practice of mathematics in Nepal because of the exposition of scattered mathematical materials in different collections [13].

In the history of Mathematics in Bharatavarsa, specially Nepal 1986 is consider as the most valuable year because of Naya Raj Pant had written a valuable treatise 'Declaration of Lichhavi Era'. This is a single treatise that declared the Lichhavi Era and this declaration is the most authorized source for the time line of Lichhavi period. This treatise was getting the *Madan Prize* which is consider as the Nobel Prize in context of Nepal. He had used many symbols to represent the page numbers of this treatise and he said, *For the entrance of the interested person to Lichhavi Numbers System and somewhat to be secure about these numbers, in this treatises' prefaces' pages be given in Lichhavi numbers* [Appendix-A].

In Changu Narayan Temple at Bhaktapur of Nepal, there were used ancient numerals symbols as the ancient records but this was not exposed abroad. These numeral symbols expressed the numbers. This numeral system links to Brahmi numerals systems. Brahmi numerals system is developed in 3rd century BC. At that time, they had written the number two and three digit in following way which shows the (+) operation system to form the new numbers [15].

$$\begin{array}{c} \text{२} + \text{९} = 13, \text{१३} + \text{१} = 28, \\ \text{१३} + \text{२} + \text{९} = 17 \end{array}$$

Fig. 3: Addition Operation and Formation of New Numbers

Numbers are seen as universal templates of creation, and therefore as symbols of perfection and of the Gods [4]. On the basis of this short preliminary review can

be concluded that the exploration of Lichhavi numeral system is relevant in the field of development of numbers, numerals and hence in mathematics.

By 200 AD, Buddhism had vanished, and was replaced by Hinduism, brought by the Lichhavi, who attacked and overthrew the last *Kirats* King Gasti. He was the 29th king of Kirats in Nepal. He ruled 85 years [20]. Altogether 1225 years (800BC-300CE) was ruled by Kirats. In the chronicle of *Vamshavalis*, the scholar William Kirk Patrick had mentioned that the Kirats dynasty was existed from about 900 BC to 300 CE. During this long period altogether 29 Kirats Kings ruled over the country [21].

The evidence for the development of Lichhavi dynasty is clarify the explanations given by Lielukhine D. N. [9] in his paper The Rise of the Lichhavi Kingdom in Nepal from the evidences of “*Gopalarajavamshavali*” and inscriptions.

Thapa (2001) described in his paper ‘History of Nepalese Buddhism: From Mythological Tradition to the Lichhavi Period’ the ruling of different Kirats Kings in Lichhavi dynasties. This paper also highlights the Lichhavi period and the academic developments of this period [18].

In Nepal, different civilizations introduced the caste system which continued today that helped in a classical age of Nepalese art and architecture. The *Vamshavalis* or chronicles, the oldest of which was written during the 14th century AD, are the only fairly reliable basis for Nepal’s ancient history. The *Vamshavalis* mention the rule of several dynasties of the Gopala’s *Vamshavalis*, the Abhiras and the *Kirats* over a stretch of centuries. Lichhavi numerals were developed in the context of major historical movements in mathematics in Nepal.

The documented history of Nepal begins with the Changu Narayan at Bhaktapur inscription of King Man Deva 1st (C. 464-505 A.D.) of the Lichhavi dynasty. According to Michihiro Narita (2008), the ancient period of Nepal was started from Lichhavi period [1].

Naya Raj Pant had declared the time line of Lichhavi period 576 to 880 AD and the Lichhavi numerals were practice later period too. For the knowledge of history, main proofs are time line of era. Right or real proofs or evidences only support to right or real knowledge. So Pant mainly focused to declare the era. He has explained various practices through the available evidences (Stone inscriptions) and the ancient treatise *Sumatitantra* to determine the Lichhavi Era [Appendix-A].

He raised the four proofs found in Nepal and *Tibet-Original statement of Sumatitantra*, *The statements added later in Sumatitantra*, *the copied Harivansha’s Puspika statement* and *Tibetan scholar S-Sakya-Pan-Chen (1182-1251)’s statement* [12].

Naya Raj Pant had declared that the treatise *Sumatitantra* was written in Nepal (Kathmandu Valley) and due to this Nepal's importance was highlight in international scenario. On the basis of his declaration of the place at where Sumatitantra was written and further declaration of the Lichhavi era on the basis of this treatise conformed that the Lichhavi numerals were Nepals fundamental numeral developments [1].

3.2 Theoretical Analysis of Development of Lichhavi Numerals

Since human have number senses. Sense of numbers mean, understands their relationship to one another, is able to perform mental math, understands symbolic representations, and can use those numbers in real world situations. The following five components that characterize number sense: Number meaning, Number relationships, Number magnitude, Operations involving numbers and referents for number, Referents for numbers and quantities [11]. In every context of human life the counting seems to be necessary and for counting need the followings advanced skills with one-to-one correspondence: Assign a rank to each object, Associate each object with the memory of all those considered before, Convert succession into simultaneity (Piano's Axioms). These characters were hold in Lichhavi numeral system [Appendix-A].

For counting we should have the symbolic concept is essentially connected with the use of a number-variable to express the form of the result of performing an arithmetical operation [16]. The numbers and numerals supports to interpret the main theme and it enhance the assimilation to the reader in any subjects' concept. Now days we used the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 to do any calculations or mathematical works. But this situation was not in ancient period. In Babylonian, Chinese, Egypt, Greek, Mayan, Roman civilization there were not used the symbols zero and they used distinct symbols to denote the separate numbers [12].

In context of Indian Subcontinental (*Bharatavarsa*) civilization, in Brahmi inscription and derived letters of this inscription and Kharosthi inscription, there were not used the symbol of zero. They used separate symbol to denote the different numbers. On the basis of archaeological records, Lichhavi era is as the Nepal's first recorded era. In this period also there were no use of symbols of zero. In this era they used different symbols to denote the each number [12].

In context of Nepal, Murray *Samrat Ashoka's* two archaeological records were the very oldest records. His these two oldest archaeological records were found at *Lumbini* of *Rupandehi* district and *Niglihawa* of *Kapilvastu* district [12]. Among them the first written record proved Lumbini as the birth place of Lord *Gautama Buddha* and the second written record gives the knowledge of existence of

Kanakmuni Buddha. In the written record of *Niglihawa* the word *Chodasavasabhisiten* and both of these two record included the word *Visativasabhisiten*. At the time of twentieth year's period coronation installation of throne of *Ashoka* used these two word to denote the 'fourteen' and 'twenty' in *Brahmi* inscriptions in Nepali language in both of the *Ashoka* pillar at *Lumbini* [12].

3.3 Symbolization of Lichhavi Numeral System

A scholar Bhagwanlal Indraji in Gujrat of India published 15 written records of Lichhavi period in 1880. In these records the first written record is *Changunarayan* Man Deva pillar of *Sambat* 386. In this record three symbols 𑒧𑒫𑒛𑒫 were used for the number "Three hundreds Eighty Six 386" [12]. All the other numerals were not possible to express here. In the later period there were other older written records than *Changunarayan* were found. From the archaeological evidences *Maligaun's Jay Varma's* *Sambat* 107 was became the first written record. To denote the number 107, used two symbols 𑒧𑒫 [12].

Further these records help the development of other numeral systems and mathematical developments. By observation we can see in the following table that consists the 17 Lichhavi numerals. Another one symbol is used for 1000, so altogether there were 18 numeric symbols. They used ligatures after each hundred places are the most popular feature in this numeral system. The symbol for 1000 is illustrated below. The repeated symbol is counted once. The symbols used for 1, 2, 3 are alike and the symbols used to denote 100, 200 and 300, etc. are alike. There is repetitions of same symbols or ligatures.



Fig. 4: Lichhavi Symbols for 1000

Tab. no.2: Lichhavi Numeral Symbols-Modified by Ram Man Shreshtha [14]

𑒧	𑒨	𑒩	𑒪	𑒫	𑒬	𑒭	𑒮	𑒯
1	2	3	4	5	6	7	8	9
𑒰	𑒱	𑒲	𑒳	𑒴	𑒵	𑒶	𑒷	𑒸
10	20	30	40	50	60	70	80	90
𑒹	𑒺	𑒻	𑒼	𑒽	𑒾	𑒿	𑓀	𑓁
100	200	300	400	500	600	700	800	900

3.4 Pattern of Lichhavi Numeral System

At Lichhavi period's record the symbol of Zero was not used. To denote the numbers from one (1) to nine (9) different symbols were used. Similarly different symbols were used to denote the numbers 10, 20 to 90. By the same way different symbols were used to denote the numbers 100, 200, 300, 400, 500, 600. By using the symbols of 10 and 1 they represent the number 11. By using the symbols of 10 and 2 they represent the number 12. The number 21 was represented by using the symbols of 20 and 1. The number 49 was represented by using the symbol of 40 and 9. Like manner 427 was represented by using the symbol of 400, symbols of 20 and the symbol of 7 respectively. In this way Lichhavi numerals system were developed.

In Lichhavi period there were no symbol of zero but the concept of zero was raised as the emptiness. In this way most of the concept of zero expressed emptiness either using zero or not. Boucenna expressed his views about the concept of zero [2]. Nepali or Lichhavi number system based on place value system as Hindu Arabic numeral system and Brahmi numeral system.

Now, it is explained four ligatures were used in 100 to denote 400. Five ligatures in 100 to denote 500 and six ligatures in 100 for 600. The notation  for 500 is found at Deupatan's Ratnasangha's Sambat 399. The symbol is used for 500 [12]. Different evidenced showed that in Lichhavi period the symbols were used to denote up to 600 and on the basis of this symbol we can write up to 699. But the symbol for 700 were not found in Lichhavi periods records, so we can say how the symbols were used for more than 700. Rather than this it is clearly claim that between the numbers 400 to 600, people were used the four ligatures, five ligatures, and six ligatures with 100 to form the numbers 400, 500 and 600. On the pattern or basis of these numbers construction can be seven (7) ligatures, eight (8) ligatures and nine (9) ligatures would be used to form the numbers 700, 800 and 900. On these basis we can be write up to 999.

In the symbol of 100 they were used two ligatures in right side to denote the number 200. Similarly two ligatures were used to denote the number 300. In this context there is a confusion between the symbols for 200 and 300. By using 100's symbol and the symbol of 4 were connected by a line to denote the number 400. By the same manner by using 100s symbol and the symbol of 5 were connected by a line to denote the number 500. In the same pattern the symbols up to 900 were developed. On the basis of this pattern by adding ligatures on the right side of 1000 can be formed 2000, 3000 and so on. By the same way by adding ligatures to 4, 5, 6, 7, 8 and 9 can be formed up to 9000 number. On the basis of these pattern by adding

the symbol between 10 to the symbols of 90 to the 1000 can be formed 99999.

Such symbols were not found in outside of Nepal. Ranjana numeral and Brahmi numerals were also in practice in Nepal [17].

4. Conclusions

The Lichhavi numerals were used in 3rd century BC and practiced up to 10th century AD. There were not found other reliable evidences to declare the time line of Lichhavi period and no one denies was existed to the Pants' declaration of Lichhavi period. So on the basis of Naya Raj Pants' analysis the time line of Lichhavi period is 576 to 880 AD and it was practiced later periods too. Lichhavi numeral system consist 18 numeric symbols. It is Based in base ten, non-positional system with unary, using ligatures to denote the 100 and multiple of 100 and 1000. It has no symbol of zero but at that period the concept of zero was developed. Due to absence of zero no hurdle in the system, it has used the numeric symbols in left to right decreasing magnitude order. It has additive grouping system. It followed the theory of one-to-one and the nature of numbers and counting principles with Piano's Axioms for succession of numbers.

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Appendix-A

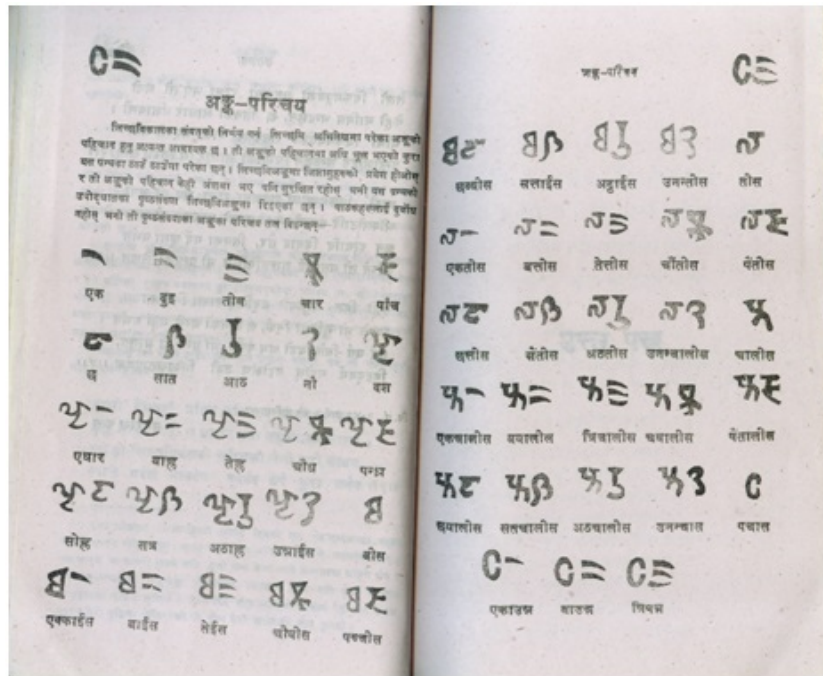


Fig. 5: A Leaf of the book Declaration of Lichhavi Era Pant, N. R. (1986). Declaration of Lichhavi Era, Kathmandu: Royal Nepal Academy, Kamaladi.

